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# Vector Boson Elastic Scattering and Compton Scattering[1]

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*Abstract*

The most general Lorentz covariant scattering amplitude is constructed for the elastic scattering of neutral vector bosons and pions, using a symmetric spinor description for the vector particles. In the limit of zero mass for the vector boson, the known results for Compton scattering are reproduced in a simplified way that avoids discussion of gauge invariance. In this limit the scattering is generalized from photons to any massless particle. Finally, a brief discussion is given of the single-particle pole terms in Compton scattering.

## 1. *Introduction*

The usual approach (see, for example, Hearn, 1961; Bardeen & Tung, 1968; and Conway, 1968) to the problem of constructing the most general scattering amplitude, consistent with Lorentz invariance and invariance with respect to the appropriate discrete symmetries, for processes involving particles with spin is to form a set of basis vectors from the momenta, and to make invariants from them and the spin functions that are available, i.e., gamma matrices and Dirac bispinors for spin-$\frac{1}{2}$, polarization vectors for spin-1, and so on for higher spins. Particularly when some of the particles are massless, this method has difficulties because of the necessity of imposing gauge invariance for spin-1 massless particles and similar conditions for higher-spin massless particles.

The purpose of this paper is to construct the scattering amplitude, consistent with Lorentz invariance and invariance with respect to space and time reflection and charge conjugation, for the elastic scattering of neutral vector mesons and pions. In contrast to previous treatments of this case (see, for example, Ebata & Lassila, 1969; Müller & Vahedi-Faridi, 1973) the vector mesons will be represented by symmetric spinor field operators. This simplifies the problem, particularly the massless limit when the vector bosons become photons, because gauge invariance is automatically satisfied by the symmetric spinors and need not be imposed as an additional constraint. This point has been particularly emphasized by Zwanziger (1964, 1965). After constructing

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the general scattering amplitude, the behavior of the invariants will be examined as the vector meson masses go to zero, and in this limit of Compton scattering, the scattering process will be generalized from photons to any massless particle. Finally, for Compton scattering, the single-particle pole terms will be exhibited and shown to be independently gauge invariant, a point of view that differs from some discussions of the problem.

## 2. *Invariant Amplitude*

For the process

$$V_1(K_1) + \pi(Q_1) \to V_2(K_2) + \pi(Q_2)$$

let the four-momentum operator $-i\,\partial/\partial x_\mu$ (in units $\hbar = c = 1$) be written as $K_1$ when it operates on the $V_1$ field operator, $K_2$ on $V_2$, etc. so that the order of derivative factors may be disregarded. The problem, then, is to construct the most general Lorentz invariant $\mathscr{R}$ operator, defined in terms of the scattering operator $S$ by

$$S = 1 + i \int d^4x \mathscr{R}(x)$$

using only the symmetric spinor field operators and their derivatives, subject to the spinor wave equations. Invariance of the $S$ operator to space-time translations leads to four-momentum conservation for the matrix elements. Consequently, only three of the field operators have independent derivatives. They are chosen to be $K_1$, $K_2$ and $(Q_1 + Q_2)/2 \equiv Q$. In the usual spinor notation[2] a vector meson is described by a symmetric spinor pair $\chi_{\dot{\alpha}_1\dot{\alpha}_2}(x)$, $\varphi^{\alpha_1\alpha_2}(x)$ coupled by a second-order wave equation. The symmetry makes the number of independent components three, appropriate for a spin-1 particle. A pseudoscalar particle is described by a one-component field operator $\phi(x)$.

In detail, the Lorentz covariant, space-inversion, translation and charge conjugation covariant $\mathscr{R}$ operator suitable for the direct reaction $(V_2 \neq V_1)$ and all related processes is

$$\begin{aligned}
\mathscr{R}(x) = \{ & B_1 [\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2) + \varphi^{\alpha_1\alpha_2}(1)\chi_{\alpha_1\alpha_2}(2)] \\
& + B_2 [\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1) Q^{\gamma_1\dot{\alpha}_1} Q^{\gamma_2\dot{\alpha}_2} \chi_{\gamma_1\gamma_2}(2) + \varphi^{\alpha_1\alpha_2}(1) Q_{\alpha_1\dot{\gamma}_1} Q_{\alpha_2\dot{\gamma}_2} \varphi^{\dot{\gamma}_1\dot{\gamma}_2}(2)] \\
& + B_3 [\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1) K_1^{\gamma_1\dot{\alpha}_1} K_2^{\gamma_2\dot{\alpha}_2} \chi_{\gamma_1\gamma_2}(2) + \varphi^{\alpha_1\alpha_2}(1) K_{1\alpha_1\dot{\gamma}_1} K_{2\alpha_2\dot{\gamma}_2} \varphi^{\dot{\gamma}_1\dot{\gamma}_2}(2)] \\
& + B_4 [\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1) K_1^{\gamma_1\dot{\alpha}_1} Q^{\gamma_2\dot{\alpha}_2} \chi_{\gamma_1\gamma_2}(2) + \varphi^{\alpha_1\alpha_2}(1) K_{1\alpha_1\dot{\gamma}_1} Q_{\alpha_2\dot{\gamma}_2} \varphi^{\dot{\gamma}_1\dot{\gamma}_2}(2)] \\
& + B_5 [\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1) K_2^{\gamma_1\dot{\alpha}_1} Q^{\gamma_2\dot{\alpha}_2} \chi_{\gamma_1\gamma_2}(2) \\
& + \varphi^{\alpha_1\alpha_2}(1) K_{2\alpha_2\dot{\gamma}_2} Q_{\alpha_2\dot{\gamma}_2} \varphi^{\dot{\gamma}_1\dot{\gamma}_2}(2)] \} \phi\phi^\dagger \\
& + \text{charge conjugate}
\end{aligned}$$

[2] See, for example, Weaver & Fradkin (1965).

The terms in the { } brackets are understood to be symmetrized in the usual way,[3] and the amplitudes $B_i$ are scalar functions of the independent derivatives. Schematically, the $\mathscr{R}$ operator may be written

$$\mathscr{R} = \sum_{i=1}^{5} B_i [N_i + N_i^c] \phi\phi^\dagger$$

with $N_i^c$ and $N_i$ being charge conjugates. If the initial and final vector mesons are identical, then $N_i^c = N_i$ for $i = 1, 2, 3$ and $N_4^c = N_5, N_5^c = N_4$ so that only four of the five invariants remain independent. Rewriting $\mathscr{R}$ for this case gives

$$\mathscr{R} = \sum_{i=1}^{4} A_i M_i \phi\phi^\dagger$$

where

$$M_1 = \{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2) + \varphi^{\dot{\alpha}_1\dot{\alpha}_1}(1)\chi_{\alpha_1\alpha_2}(2)\}$$

$$M_2 = \{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)Q^{\gamma_1\dot{\alpha}_1}Q^{\gamma_2\dot{\alpha}_2}\chi_{\gamma_1\gamma_2}(2) + \varphi^{\gamma_1\gamma_2}(1)Q_{\gamma_1\dot{\alpha}_1}Q_{\gamma_2\dot{\alpha}_2}\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)\}$$

$$M_3 = \{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)K_1^{\gamma_1\dot{\alpha}_1}K_2^{\gamma_2\dot{\alpha}_2}\chi_{\gamma_1\gamma_2}(2) + \varphi^{\gamma_1\gamma_2}(1)K_{1\gamma_1\dot{\alpha}_1}K_{2\gamma_2\dot{\alpha}_2}\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)\}$$

$$M_4 = \{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)(K_1 + K_2)^{\gamma_1\dot{\alpha}_1}Q^{\gamma_2\dot{\alpha}_2}\chi_{\gamma_1\gamma_2}(2) + \varphi^{\gamma_1\gamma_2}(1)(K_1 + K_2)_{\gamma_1\dot{\alpha}_1}Q_{\gamma_2\dot{\alpha}_2}\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)\}$$

The matrix elements of $\mathscr{R}$, integrated over all space-time are proportional to the usual $T$ matrix elements, and one finds, for example, the crossing properties of the $A_i$ by the standard techniques, i.e., if the vector meson is self-conjugate, then comparison of the two matrix elements

$$\int d^4x \langle \pi(Q_2)V(K_2) | \mathscr{R} | \pi(Q_1)V(K_1) \rangle$$

and

$$\int d^4x \langle \pi(Q_2)V(-K_1) | \mathscr{R} | \pi(Q_1)V(-K_2) \rangle$$

yields the crossing properties.

### 3. *Massless Limit*

To study the $\mathscr{R}$ operator as a function of the vector meson mass, one must look in detail at the field operators. They are

$$\chi_{\dot{\alpha}_1\dot{\alpha}_2}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3P}{\sqrt{2E}} \frac{[E + m_V + \boldsymbol{\sigma}\cdot\mathbf{P}]_{\alpha_1\beta_1}}{2(E + m_V)} [E + m_V + \boldsymbol{\sigma}\cdot\mathbf{P}]_{\alpha_2\beta_2} \times \sum_{k=-1}^{1} \mathscr{U}_{\beta_1\beta_2}(\hat{\mathbf{P}}, k)[a(\mathbf{P}, k)e^{iPx} + (-1)^{1-k}a^\dagger(\mathbf{P}, -k)e^{-iPx}]$$

[3] See, for example, Sakurai (1964).

and a similar expression for $\varphi^{\alpha_1\alpha_2}(x)$ with $\boldsymbol{\sigma}\cdot\mathbf{P}$ replaced by $-\boldsymbol{\sigma}\cdot\mathbf{P}$. Here $k$ is the polarization quantum number, the $\mathscr{U}(\mathbf{P}, k)$ are the momentum spinors appropriate for spin-1[4] and $a(\mathbf{P}, k)$, $a^\dagger(\mathbf{P}, k)$ are the destruction and creation operators. There are two types of functions to consider as $m_V$ goes to zero. First,

$$\chi_{\dot{\alpha}_1\dot{\alpha}_2}(x)\xrightarrow{m_V\to 0}\frac{1}{(2\pi)^{3/2}}\int d^3P(2P)^{1/2}\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, 1)[a(\mathbf{P}, 1)e^{iPx}+a^\dagger(\mathbf{P},-1)e^{-iPx}]$$

$$\varphi^{\alpha_1\alpha_2}(x)\xrightarrow{m_V\to 0}\frac{1}{(2\pi)^{3/2}}\int d^3P(2P)^{1/2}\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, -1)[a(\mathbf{P}, -1)e^{iPx}+a^\dagger(\mathbf{P}, 1)e^{-iPx}]$$

because $[P\pm\boldsymbol{\sigma}\cdot\mathbf{P}]_{\alpha_1\beta_1}[P\pm\boldsymbol{\sigma}\cdot\mathbf{P}]_{\alpha_2\beta_2}\mathscr{U}_{\beta_1\beta_2}(\mathbf{P}, k)=0$ unless $k=\pm 1$, respectively. So the symmetric spinor field operators go smoothly to the massless limit appropriate for the photon. Second, the derivatives of the vector meson field operators are proportional to the mass of the vector meson and so go smoothly to zero in the massless limit. In detail, the term $K_1^{\gamma_1\dot{\alpha}_1}\chi_{\dot{\alpha}_1\dot{\alpha}_2}(x)$ is

$$\begin{aligned}
K_1^{\gamma_1\dot{\alpha}_1}\chi_{\dot{\alpha}_1\dot{\alpha}_2}(x) &= m_V\left\{\frac{1}{(2\pi)^{3/2}}\int\frac{d^3P}{\sqrt{2E}}\frac{(E+m_V)}{2}\left[1-\frac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+m_V}\right]_{\gamma_1\beta_1}\right.\\
&\quad\times\left[1+\frac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+m_V}\right]_{\alpha_2\beta_2}\sum_{k=-1}^{1}\mathscr{U}_{\beta_1\beta_2}(\hat{\mathbf{P}}, k)[a(\mathbf{P}, k)\,e^{iPx}\\
&\quad\left.+(-1)^k a^\dagger(\mathbf{P}, -k)\,e^{-iPx}]\right\}\xrightarrow{m_V\to 0} m_V\left\{\frac{1}{(2\pi)^{3/2}}\int\frac{d^3P}{2}\sqrt{\frac{P}{2}}\right.\\
&\quad\times[1-\boldsymbol{\sigma}\cdot\hat{\mathbf{P}}]_{\gamma_1\beta_1}[1+\boldsymbol{\sigma}\cdot\hat{\mathbf{P}}]_{\alpha_2\beta_2}\mathscr{U}_{\beta_1\beta_2}(\mathbf{P}, 0)[a(\mathbf{P}, 0)\,e^{iPx}\\
&\quad\left.+a^\dagger(\mathbf{P}, 0)\,e^{-iPx}]\right\}
\end{aligned}$$

which goes smoothly to zero. So, as the vector meson mass goes to zero, the invariant $M_3$ vanishes as $m_V^2$ and $M_4$ vanishes as $m_V$. There are no relations that the invariant amplitudes $A_i$ are required to satisfy in this limit, in contrast to the usual method. The resulting $\mathscr{R}$ operator for Compton scattering is

$$\begin{aligned}
\mathscr{R}(x) = \{&A_1[\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)+\varphi^{\alpha_1\alpha_2}(1)\chi_{\alpha_1\alpha_2}(2)]\\
&+A_2[\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)Q^{\gamma_1\dot{\alpha}_1}Q^{\gamma_2\dot{\alpha}_2}\chi_{\gamma_1\gamma_2}(2)+\varphi^{\gamma_1\gamma_2}(1)Q_{\gamma_1\dot{\alpha}_1}Q_{\gamma_2\dot{\alpha}_2}\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)]\}\phi\phi^\dagger
\end{aligned}$$

to make the connection with the usual tensor formulation (Bardeen & Tung, 1968) one notes that

$$\{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)+\varphi^{\alpha_1\alpha_2}(1)\chi_{\alpha_1\alpha_2}(2)\}\,\phi\phi^\dagger\sim F_{\mu\nu}(1)F_{\mu\nu}(2)\phi\phi^\dagger$$

$$\{\chi_{\dot{\alpha}_1\dot{\alpha}_2}(1)Q^{\gamma_1\dot{\alpha}_1}Q^{\gamma_2\dot{\alpha}_2}\chi_{\gamma_1\gamma_2}(2)+\varphi^{\gamma_1\gamma_2}(1)Q_{\gamma_1\dot{\alpha}_1}Q_{\gamma_2\dot{\alpha}_2}\varphi^{\dot{\alpha}_1\dot{\alpha}_2}(2)\}\,\phi\phi^\dagger \sim F_{\mu\nu}(1)F_{\rho\mu}(2)Q_\rho Q_\nu\phi\phi^\dagger$$

[4] See the Appendix for a detailed discussion.

By using the symmetric spinor formulation one has dealt directly with the gauge invariant combination of electric and magnetic fields.[5]

The above $\mathscr{R}$ operator for Compton scattering can be easily generalized to the elastic scattering of spin-$S$ massless particles from the same target. The result is[6]

$$\begin{aligned}\mathscr{R}(x) = \{&A_1 [\chi_{\dot\alpha_1 \dots \dot\alpha_2 S}(1)\varphi^{\dot\alpha_1 \cdots \dot\alpha_2 S}(2) + \varphi^{\dot\alpha_1 \cdots \dot\alpha_2 S}(1)\chi_{\alpha_1 \dots \alpha_2 S}(2)]\\ &+ A_2 [\chi_{\dot\alpha_1 \dots \dot\alpha_2 S}(1) Q^{\gamma_1 \dot\alpha_1} \cdots Q^{\gamma_2 S \dot\alpha_2 S} \chi_{\gamma_1 \dots \gamma_2 S}(2) + \varphi^{\gamma_1 \cdots \gamma_2 S}(1) Q_{\gamma_1 \dot\alpha_1} \cdots\\ &\times Q_{\gamma_2 S \dot\alpha_2 S}\, \varphi^{\dot\alpha_1 \cdots \dot\alpha_2 S}(2)]\}\phi\phi^\dagger\end{aligned}$$

## 4. *Pole Contributions*

Once the most general $\mathscr{R}$ operator for Compton scattering has been derived and the appropriate matrix element taken, the question of determining the invariant amplitudes $A_i$ arises. Generalized unitarity supplies information about $A_i$ and, in particular, it leads one to expect contributions from single-particle exchanges in the various channels. In an $S$-matrix theory one finds the exchange contributions to the invariant amplitudes following the Cutkosky (1960) procedure, and then expands the resulting Lorentz invariants in terms of the previously found $M$ functions, the coefficients being the contributions to the $A_i$.

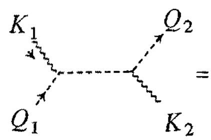
Neglecting unessential details, generalized unitary tells one that in the $s$ channel (direct channel) the single-pion exchange contributes to the scattering amplitude the term[7]

$$4e^2 \epsilon_1 \cdot Q_1 \epsilon_2 \cdot Q_2\, \delta(s - \mu^2)$$

which may be rewritten as

$$\begin{aligned}4e^2 \epsilon_1 \cdot Q_1 \epsilon_2 \cdot Q_2\, \delta(s - \mu^2) = e^2 \{&-F_{\mu\nu}(1)F_{\mu\nu}(2)/2\\ &+ [8/(u - \mu^2)]\, F_{\mu\nu}(1)F_{\rho\mu}(2)Q_\rho Q_\nu\\ &+ (s - \mu^2)C\}\, \delta(s - \mu^2)\end{aligned}$$

Note that at $s = \mu^2$, $K_1 \cdot Q_1 = K_2 \cdot Q_2 = 0$. Since one wants the *residue* at the position $s = \mu^2$ the term $(s - \mu^2)C$ does not contribute and the final result for the $s$-channel pion *pole* is



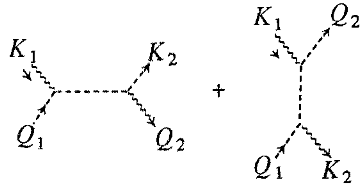

$$= \frac{e^2}{2(s - \mu^2)} F_{\mu\nu}(1)F_{\mu\nu}(2) - \frac{8e^2}{(s - \mu^2)(u - \mu^2)} F_{\mu\nu}(1)F_{\rho\mu}(2)Q_\rho Q_\nu$$

[5] This point is amplified in the Appendix.

[6] A symmetric spinor with $2S$ indices has $2S + 1$ independent components and is appropriate for describing a spin-$S$ particle.

[7] $\epsilon_{1,2}$ are the polarization vectors of the incident and scattered photons, and $s$ and $u$ are the usual Mandelstam variables. The pion has charge $e$ and mass $\mu$.

A similar calculation gives the contribution of the $u$-channel (crossed channel) pion pole or one may use the crossing symmetry of the amplitudes: The result is the same in either case, the final result being




$$= \frac{e^2}{2}\left(\frac{1}{s-\mu^2} + \frac{1}{u-\mu^2}\right) F_{\mu\nu}(1)F_{\mu\nu}(2)$$

$$- \frac{16e^2}{(s-\mu^2)(u-\mu^2)} F_{\mu\nu}(1)F_{\rho\mu}(2)Q_\rho Q_\nu$$

so the pion exchange leads to products of singularities. This kind of singularity structure occurs when the exchanged particle and the identical external particle coupled with the photon via a charge (monopole) coupling, and, in fact, for this case, all processes involving photons have such singularities. A final point to note is that care must always be taken to distinguish between the Born approximation and pole contributions to the generalized unitarity relations (see, for example, Berends et al., 1967).

## *Appendix*

The purpose of this Appendix is to give some details of the momentum space spinors $\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, k)$, and the relation between symmetric spinor field operators and the electric and magnetic fields of the photon.

The momentum spinors may be written as

$$\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, 1) = \mathscr{U}_{\alpha_1}(\hat{\mathbf{P}}, \tfrac{1}{2})\mathscr{U}_{\alpha_2}(\hat{\mathbf{P}}, \tfrac{1}{2})$$

$$\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, 0) = (1/\sqrt{2})[\mathscr{U}_{\alpha_1}(\hat{\mathbf{P}}, \tfrac{1}{2})\mathscr{U}_{\alpha_2}(\hat{\mathbf{P}}, -\tfrac{1}{2}) + \mathscr{U}_{\alpha_1}(\hat{\mathbf{P}}, -\tfrac{1}{2})\mathscr{U}_{\alpha_2}(\hat{\mathbf{P}}, \tfrac{1}{2})]$$

$$\mathscr{U}_{\alpha_1\alpha_2}(\hat{\mathbf{P}}, 0) = \mathscr{U}_{\alpha_1}(\hat{\mathbf{P}}, -\tfrac{1}{2})\mathscr{U}_{\alpha_2}(\hat{\mathbf{P}}, -\tfrac{1}{2})$$

where

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{P}} \quad (\hat{\mathbf{P}}, \pm\tfrac{1}{2}) = \pm \quad (\hat{\mathbf{P}}, \pm\tfrac{1}{2})$$

If $\theta$ and $\phi$ are the polar and azimuthal angles of $\hat{\mathbf{P}}$, then a specific representation is

$$\mathscr{U}(\hat{\mathbf{P}}, \tfrac{1}{2}) = \begin{bmatrix} \cos\tfrac{1}{2}\theta \\ \sin\tfrac{1}{2}\theta\, e^{i\phi} \end{bmatrix}, \qquad \mathscr{U}(\hat{\mathbf{P}}, -\tfrac{1}{2}) = \begin{bmatrix} -\sin\tfrac{1}{2}\theta\, e^{-i\phi} \\ \cos\tfrac{1}{2}\theta \end{bmatrix}$$

To make the connection between symmetric spinors and the electric and magnetic fields of the photon, one notes (Good, 1957; Haji & Weaver, 1969) that Maxwell's equations fcr the photon can be written as the matrix equation

$$\overline{\mathbf{S}}\cdot\mathbf{P}\overline{\psi}(x) = i(\partial/\partial t)\overline{\psi}(x)$$

and auxiliary condition

$$(\partial/\partial x_k)\bar{\psi}_k(x) = 0, \qquad (\bar{S}_i)_{\partial k} = -i\epsilon_{ijk}$$

where $\bar{\psi}$ is a three-component wavefunction with elements

$$\bar{\psi}_k = E_k + iB_k$$

Then, the unitary transformation

$$\psi = U\bar{\psi}$$

where

$$U = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{bmatrix}$$

and

$$\mathbf{S} = U\bar{\mathbf{S}}U^\dagger$$

take the theory to the representation in which

$$\psi_1 = \chi_{11}, \qquad \psi_2 = \sqrt{2}\chi_{\dot{2}1} = \sqrt{2}\chi_{1\dot{2}}, \qquad \psi_3 = \chi_{22}$$

A similar analysis connects the combination $\mathbf{E} - i\mathbf{B}$ and the spinor functions $\varphi^{\alpha_1\alpha_2}$. Finally, symmetric spinors are related to the antisymmetric tensor combination of $\mathbf{E}$ and $\mathbf{B}$ according to

$$\chi_{\dot{\alpha}_1\dot{\alpha}_2} \sim (i\sigma_\mu^\dagger \sigma_\nu \sigma_2)_{\alpha_1\alpha_2} F_{\mu\nu}$$

$$\varphi^{\alpha_1\alpha_2} \sim (-i\sigma_\mu \sigma_\nu^\dagger \sigma_2)_{\alpha_1\alpha_2} F_{\mu\nu}$$

where $\sigma_4$ is $i$ times the $2 \times 2$ identity matrix, another way of obtaining the connection.

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